NON-UNIVERSAL SUSY BREAKING, HIERARCHY AND SQUARK DEGENERACY[*][†]

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ABSTRACT

I discuss non-trivial effects in the soft SUSY breaking terms which appear when one integrates out heavy fields. The effects exist only when the SUSY breaking terms are non-universal. They may spoil (1) the hierarchy between the weak and high-energy scales, or (2) degeneracy among the squark masses even in the presense of a horizontal symmetry. I argue, in the end, that such new effects may be useful in probing physics at high-energy scales from TeV-scale experiments.

Disclaimer

1. Introduction

The Standard Model (SM) of the particle physics is extremely succesfull and is now being tested experimentally at a precision better than 1 % level. However, it leaves many questions unanswered: the origin of flavor, rather complex quantum numbers under the gauge group, anomaly cancellation, charge quantization, and many others. Any attemps to build models which answer these questions involve new physics at much deeper levels, *e.g.*, much higher energies. Then one has to ensure that the hierarchy between the weak scale and the energy scales of new physics is stable under the radiative corrections. Supersymmetry (SUSY) has been regarded as a promising candidate to ensure the stability of such a hierarchy.

SUSY, however, has also many problems especially from the model building point of view. First of all, there is no concensus how the supersymmetry is broken. It tends to give too large rates for the flavor-changing neutral current processes. And, the most importantly, supersymmetry itself does not explain the hierarchy; it merely stabilizes it. For a more complete list of the problems, I refer to a talk by Haber.[1]

In this talk, I point out several other problems in SUSY model building which, to my understanding, are not widely recognized; these problems arise only when the SUSY breaking terms are non-universal. The first is that the hierarchy may be spoiled by the SUSY breaking effect. The second is that the degeneracy among the scalar quarks may not be guaranteed even with the horizontal symmetries. Both of the problems can be discussed within the same context: integrating out heavy fields in the presence of the SUSY breaking effects. Integrating out the heavy fields is not the same as throwing them away; they leave non-trivial relics in the soft SUSY breaking terms in the low-energy effective theory. I will exemplify how non-trivial effects arise in the next few sections.

Let me remind you that having many heavy fields at a mass scale M below the Planck scale M_P is a relatively generic feature of the SUSY models. SUSY GUT of course have many heavy fields at the GUT-scale $M \simeq 10^{16}$ GeV, and they have to be integrated out. Most of the flavor models also have many heavy fields below the Planck scale; one uses $M/M_P \sim 0.01$–0.1 as a small expansion parameter to reproduce the hierarchical structure of the Yukawa matrices. Therefore, it is a very general question to analyze the soft SUSY breaking terms when you integrate out heavy fields.

2. Naive Integration of Heavy Fields

Let me first explain what kind of misconception I myself had in the past.[a]

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[a]All the discussion applies only to the framework where the SUSY-breaking masses are fed into the

Suppose we have a SUSY model with superheavy fields, *e.g.*, at the scale of the grand unified theory (GUT). When we derive the Minimal Supersymmetric Standard Model (MSSM) from a typical SUSY GUT, we have to integrate out the superheavy fields to obtain the MSSM as an effective low-energy theory. Of course the superpotential of the model has to be chosen such that the doublet Higgs superfields in the MSSM have masses only of $O(m_W)$, either by a fine-tuning or some other "natural" mechanisms. *So far it is completely true.*

When we integrate out the heavy fields, the SUSY breaking effects are negligible, since they are much smaller than the physics scale under discussions, *e.g.*, $m_{SUSY} \ll M_{GUT}$. Therefore, we can integrate out the heavy fields without the SUSY breaking effects in mind, and write down the SUSY Lagrangian of the MSSM. Then we introduce SUSY breaking terms later, at $O(m_{SUSY})$. The SUSY breaking terms in the MSSM satisfy boundary conditions dictated by the symmetries of the original theory, such as GUT symmetry or horizontal symmetries. For instance, an $SU(2)$ horizontal symmetry between the first and second generations guarantees $m_{\tilde{d}} = m_{\tilde{s}}$. *This is completely wrong.*

There are two main mistakes in the results we obtain under this "naive" integration of the heavy fields. First, the "light" fields in the low-energy theory may have SUSY-breaking mass terms which are much bigger than $O(m_{SUSY}^2)$, thereby spoiling the hierarchy. Second, the SUSY-breaking masses in the low-energy theory may not respect the symmetries in the original theory at all. These are the points which I'll explain in this talk. Although these cases are problematic, it is welcome in general to have effects of the heavy fields in the soft SUSY breaking terms of the light fields. What we learn here is that the SUSY-breaking masses are much more sensitive to the physics at very high energy scales than we naively think. This opens up a wider "window" to the physics at very high energy scales for us.[3]

3. General Soft SUSY Breaking Terms

Under the popular assumption of the "minimal supergravity" (or "universal" SUSY breaking terms), the soft SUSY breaking terms take the following form:

$$V_{SUSY} = \mathcal{A}W + \mathcal{B}z^i F_i^* + h.c., \tag{1}$$

where W is the superpotential, F^i is the auxiliary component of the chiral supermultiplet whose scalar component is z^i, and $\mathcal{A}$, $\mathcal{B}$ are dimensionful parameters of $O(m_{SUSY})$. This form may look unfamliar, but it should look familiar after integrating out the auxiliary fields:

$$V_{SUSY} = (\mathcal{A} + 3\mathcal{B})W_3 + (\mathcal{A} + 2\mathcal{B})W_2 + (\mathcal{A} + \mathcal{B})W_1 + h.c. + \mathcal{B}^2 |z^i|^2. \tag{2}$$

fields of our interest at a scale above the scale of the heavy fields which we integrate out, *e.g.*, hidden sector models. If the SUSY breaking effects appear at very low-energy,[2] the problems may not exist.

Here, W_3 contains trilinear terms in the superpotential W, W_2 bilinear, and W_1 linear. Actually, one can prove that the "naive" integration of the heavy fields explained in the previous section is exact up to a redefinition of the $\mathcal{A}$ and $\mathcal{B}$ parameters; this amazing result was derived by Hall, Lykken and Weinberg[4] a decade ago.

The most general soft SUSY breaking terms can be written as follows if one does not assume the "universality,"

$$V_{SUSY} = \mathcal{A}_m W_m + \mathcal{B}_i^j z^i F_j^* + h.c. + (m^2)_i^j z^i z_j^*, \tag{3}$$

where W_m refers to invidual terms in the superpotential with arbitrary independent SUSY breaking coefficients $\mathcal{A}_m$. The parameter $\mathcal{B}$ in the universal case is extended to be an arbitrary matrix in the field space. In addition, one can add arbitrary scalar mass matrix m^2.

There are at least three reason why we want to consider non-universal SUSY breaking terms at the scale M where we integrate out heavy fields. (1) They may be non-universal already at M_P, like in superstring theories. (2) Universal SUSY breaking terms are not stable under the renormalization, and hence may be corrected by the physics at the Planck scale. (3) Their running from M_P to M spoil the universality. Therefore, we have to integrate out heavy fields in the presence of non-universal SUSY breaking terms.

4. Spoiling Hiearchy by SUSY-breaking Effects

In this section I present two examples where the fields which have only $O(m_{SUSY})$ masses in the superpotential can acquire soft SUSY breaking masses of $O(m_{SUSY} M_X)$, where M_X is the scale of the heavy fields you are integrating out.

The first one is the famous example of the minimal $SU(5)$ GUT by Dimopoulos, Georgi[5] and Sakai.[6] The $SU(5)$ symmetry is broken by an adjoint Higgs superfield Σ, and the Higgs doublets belong to $SU(5)$ quintets, H_u and H_d. The superpotential of this model is

$$W = \lambda \mathrm{tr}\Sigma^3 + M_\Sigma \mathrm{tr}\Sigma^2 + H_u(f\Sigma + M_H)H_d. \tag{4}$$

λ, f are dimensionless coupling constants, while M_Σ, M_H are GUT scale mass parameters. We add the most general SUSY breaking terms,

$$V_{SUSY} = \lambda A_\Sigma \mathrm{tr}\Sigma^3 + B_\Sigma M_\Sigma \mathrm{tr}\Sigma^2 + f A_H H_u \Sigma H_d + B_H M_H H_u H_d + O(m_{SUSY}^2), \tag{5}$$

where A_Σ, B_Σ, A_H and B_H are the SUSY breaking parameters of order m_{SUSY}. Taking $\Sigma = \mathrm{diag}(2,2,2,-3,-3)\sigma$, the minimum of the potential lies at $\sigma_0 = 2M_\Sigma/3\lambda$ in the SUSY limit, which is shifted by $\delta\sigma = (A_\Sigma - B_\Sigma)/3\lambda$ in the presence of the SUSY breaking terms. The mixing mass of the two doublet Higgs bosons[b] $m_{12}^2 H_u H_d$ is given by

$$m_{12}^2 = 3f\sigma_0(A_\Sigma - B_\Sigma - A_H + B_H) + O(m_{SUSY}^2), \tag{6}$$

[b] Hereafter H_u and H_d represent the $SU(2)_L$ doublet Higgs multiplets.

where we have used that the supersymmetric mass of the Higgs doublets is fine-tuned to be $M_H - 3f\sigma_0 = O(m_{SUSY})$ in the superpotential. Clearly for a class of the SUSY breaking parameters where the combination $A_\Sigma - B_\Sigma - A_H + B_H$ does not vanish, m_{12}^2 lies at an intermediate scale $\sim m_{SUSY} M_X$ and the gauge hierarchy is spoiled.

One may anticipate that such a problem exists only for models which have fine-tunings as this example. I would argue, however, that this problem is rather generic. For instance, such a problem may arise even without a GUT symmetry. Let us denote doublet Higgs fields in the MSSM by H_u and H_d. Suppose there is some reason that no mass term exists for H_u and H_d in the superpotetial in the absence of SUSY breaking, and also that SUSY is broken in the hidden sector by a O'Raifeartaigh sector for definiteness. Then there is a chiral superfield X in the hidden sector which has a vacuum expectation value in the F-component, $\langle X \rangle \sim \theta^2 m_{SUSY} M_P$. Since we have to generate μ-term anyway, we need a coupling as $\int d^4\theta X^* H\overline{H}/M_P$. But then we could also have a coupling $\int d^2 X H\overline{H}$, which again leads to a too-large soft SUSY breaking mass term to the Higgs bosons.

Actually, one can prove that such a problem does not occur in a slightly restricted form of the SUSY breaking terms,[7]

$$V_{SUSY} = \mathcal{A}W + \mathcal{B}_i^j z^i \frac{\partial W}{\partial z^j} + h.c. + O(m_{SUSY}^2) \text{ terms.} \tag{7}$$

In particular, one automatically obtains the relation $A_\Sigma - B_\Sigma - A_H + B_H = 0$ in the minimal $SU(5)$ from this ansatz with no other additional constraints. This ansatz for the SUSY breaking terms have two nice features: (1) non-universal enough such that the form is stable under renormalization, and (2) still restricted enough to guarantee the hierarchy. Indeed, one can derive a general formula for the soft SUSY breaking terms after integrating out heavy fields.[7]

5. How Squark Degeneracy May Be Spoiled

In this section, I present a toy model with a global horizontal $SU(2)$ symmetry. Even though the $SU(2)$ symmetry was meant to guarantee the degeneracy between the first- and second-generation squarks, it actually doesn't in this example.

Let me first explain how additional contribution (F-term contribution)[7] can be generated to the scalar mass term in the low-energy effective theory in general by integrating out a heavy field. Suppose there is a vector-like heavy fields ψ and $\overline{\psi}$ with a mass term $M\overline{\psi}\psi$, and a light chiral field ϕ which does not have a mass term. However, the heavy and light fields mix by picking up a vacuum expectation value of the field χ. The superpotential is

$$W = M\overline{\psi}\psi + g\overline{\psi}\chi\phi + \lambda \left(\frac{1}{3}\chi^3 - \frac{1}{2}v\chi^2 \right). \tag{8}$$

The vacuum is $\langle \chi \rangle = v$. The heavy and light fields ψ' and ϕ' are defined by

$$\psi' = \frac{1}{\sqrt{M^2 + (gv)^2}}(M\psi + gv\phi), \tag{9}$$

$$\phi' = \frac{1}{\sqrt{M^2 + (gv)^2}}(-gv\psi + M\phi). \tag{10}$$

Now the SUSY breaking terms have tri-linear and bi-linear terms as well as the scalar mass terms

$$V_{\text{SUSY}} = B_\psi M\overline{\psi}\psi + gA_\phi\overline{\psi}\chi\phi + \lambda\left(\frac{A_\chi}{3}\chi^3 - \frac{B_\chi}{2}v\chi^2\right) + m_\psi^2|\psi|^2 + m_{\overline{\psi}}^2|\overline{\psi}|^2 + m_\phi^2|\phi|^2 + m_\chi^2|\chi|^2. \tag{11}$$

The point is that the three different mass matrices, namely supersymmetric mass terms in the superpotential, the SUSY breaking tri-linear and bi-linear terms, and SUSY breaking scalar mass terms cannot be simultaneously diagonalized if $A_\phi \neq A_\chi$ or $B_\psi \neq B_\chi$ or $m_\phi^2 \neq m_\psi^2$. The resulting scalar mass for ϕ' after integrating out heavy fields ψ', $\overline{\psi}$ and χ is

$$m_{\phi'}^2 = \frac{M^2}{M^2 + (gv)^2}m_\phi^2 + \frac{(gv)^2}{M^2 + (gv)^2}m_\psi^2 - \frac{M^2(gv)^2}{[M^2 + (gv)^2]^2}(A_\chi - B_\chi + B_\psi - A_\phi)^2. \tag{12}$$

If all the SUSY breaking terms are universal, *i.e.* $A_\chi = A_\phi = A$, $B_\chi = B_\psi = B$, and $m_\phi^2 = m_\psi^2 = m_0^2$, it drastically simplifies to give m_0^2, and one can pretend nothing happened by integrating out heavy fields.

Now comes the toy model with a horizontal $SU(2)_H$ symmetry. Take ψ, $\overline{\psi}$ and ϕ as left-handed quark fields for the first two generations, each $SU(2)_H$ doublets. We introduce right-handed fields $D = (d_R^c, s_R^c)^T$ and $U = (u_R^c, c_R^c)^T$ as well, both $SU(2)_H$ doublets. We regard χ as 2×2 matrix which breaks $SU(2)_H$ symmetry down to nothing by its expectation value, $\langle \chi \rangle = \text{diag}(v, V)$ with $v \ll V$.[c] Assume the following superpotential

$$W = M\overline{\psi}\psi + g_1\overline{\psi}\chi\phi + h_s\psi DH, \tag{13}$$

with $M \sim V$. It generates hierarchical Yukawa coupling constants $h_d \sim h_s(v/M)$ after integrating out ψ' and $\overline{\psi}$. Before integrating out the heavy fields, the $SU(2)_H$ symmetry guarantees the same SUSY breaking masses for the both components $\tilde{d}_L$ and $\tilde{s}_L$ in ϕ. However they have different masses in the low-energy effective theory because the mass $m_{\phi'}^2$ depends on the $\langle \chi \rangle$. Therefore the squark degeneracy is broken even in the presence of a horizontal symmetry. Similarly, you also obtain similar

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[c] One can write the most general superpotential $W = \mu^2\text{Tr}\chi + M_\chi\text{Tr}\chi^2 + \lambda\text{Tr}\chi^3$, with $\mu \ll M$ and $\lambda \sim O(1)$. Then this superpotential has a vacuum with $v \sim \mu^2/M_\chi$ and $V \sim M_\chi$. χ is actually a reducible representation under $SU(2)_H$, since it contains both singlet and adjoint components.

contributions to the ϕ' mass from the up sector. And they do not commute each other, because of the Cabbibo rotation. Therefore not only you spoil the degeneracy between the first- and second-generations, but also generate off-diagonal terms such that quark and squark masses are not aligned. Such a model gives large rates for the flavor-changing neutral current processes if not dead.

The situation is even worse when the horizontal symmetry is gauged. As known in litterature,[8] there are additional contributions to the scalar masses when the rank of the gauge group is reduced by a symmetry breaking. The easiest example of this phenomenon is when $U(1)$ gauge symmetry is broken by charge $+1$ (-1) supefields H_+ (H_-). The expectation values of H_+ and H_- can differ if their SUSY breaking masses are different, $m_+^2 \neq m_-^2$:

$$\langle D \rangle = \langle |H_+|^2 - |H_-|^2 \rangle = \frac{1}{g}(m_+^2 - m_-^2), \tag{14}$$

where g is the gauge coupling constant. Then this condensation of the D-component gives contributions to the masses of light scalar fields ϕ^i,

$$V = \sum_i g\langle D \rangle Q_i |\phi^i|^2 = (m_+^2 - m_-^2) Q_i |\phi^i|^2. \tag{15}$$

Here Q_i are the $U(1)$ charges of the fields ϕ^i. Note that the final result does not depend on the gauge coupling constant; therefore one can never turn off the D-term contribution by taking the gauge coupling constant arbitrarily small.[d] The gauged horizontal symmetries give D-term contributions to the scalar masses differently to the different generations, since they have different quantum numbers under the horizontal symmetries.

6. Final Remarks

As we have seen, the integration of heavy fields leaves rather non-trivial relics to the soft SUSY breaking term in the low-energy effective theory. They could be harmful in some cases: (1) it may spoil the hierarchy, or (2) it may spoil the squark degeneracy. Even though these two cases are problematic, I would argue that it is actually welcome to have non-trivial consequence of heavy fields in the low-energy effective theory. Of course, these effects put new challenges to the model builders. However, this also means that the soft SUSY breaking terms in the low-energy effective theory are much more sensitive to the physics at very high energy scale than we naively think. They have much richer structure than the "universal" case, at least.

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[d]In the previous example with a global horizontal symmetry, one could suppress the additional contributions by taking the limit $M \to \infty$. Of course there is a certain upper bound from the requirement that the Yukawa coupling constants are not too small. Then the question becomes a numerical one.

Therefore the future measurements of the soft SUSY breaking parameters may allow us to figure out the symmetry structure at high energies, flavor physics and so on.

Recall that the GUT-relation of the gaugino masses are very good predictions of SUSY GUT. The threshold corrections at the GUT-scale do not generate large logarithms,[9] and hence only of $O(\alpha/\pi)$, as far as the gaugino masses are comparable to other soft SUSY breaking terms from the beginning. They satisfy the same relation even in the presence of intermediate symmetries.[10] Therefore GUT-relation of the gaugino masses provide us an excellent tool to test the idea of SUSY GUT, and its test is experimentally feasible.[11]

Scalar masses are more sensitive to the detail of the physics at high energies.[10] Even though the F-term contributions can in general spoil the boundary conditions of GUT symmetry,[e] we expect such effects are small enough for the first two generations to suppress the flavor-changing effects adequately. Then the mass measurements of squarks and sleptons of the first two generations at future colliders can be still used to test the symmetries at high energy scales.[10] The masses of the third generation fields and the Higgs bosons contain more information on the physics at high scales. Finally, rare flavor-changing effects,[13] CP-violating effects[14] and proton decay[15] provide us probes to the tiny effects in the scalar mass matrices from the flavor physics at high scales. If we are lucky enough to see many different kinds of signatures in the near future, we may gain insights on physics at very high energy scales.

Acknowledgements

1. H.E. Haber, "The Supersymmetric Top Ten Lists," invited talk at International Workshop on Recent Advances in the Superworld, Woodlands, TX, 13-16 Apr 1993, hep-ph/9308209.
2. See, for instance, M. Dine, A.E. Nelson and Y. Shirman, hep-ph/9408384.
3. H. Murayama, "Looking up at the GUT/Planck World," Hitoshi Murayama, Invited Talk at the ICEPP Symposium "From LEP to the Planck World," University of Tokyo, Dec 17–18, 1992, hep-ph/9402255.
4. L.J. Hall, J. Lykken and S. Weinberg, *Phys. Rev.* **D27**, 2359 (1983).
5. S. Dimopoulos and H. Georgi, *Nucl. Phys.* **B193**, 150 (1981).
6. N. Sakai, *Z. Phys.* **C11**, 153 (1981).

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[e]This point was recently emphasized by Dimopoulos and Pomarol,[12] and their results are consistent with the general formula.[7]

7. Y. Kawamura, H. Murayama, and M. Yamaguchi, *Phys. Rev.* **D51**, 1337 (1995).

8. M. Drees, *Phys. Lett.* **B181**, 279 (1986); J.S. Hagelin and S. Kelley, *Nucl. Phys.* **B342**, 95 (1990).

9. J. Hisano, H. Murayama, and T. Goto, *Phys. Rev.* **D49**, 1446 (1994).

10. Y. Kawamura, H. Murayama, and M. Yamaguchi, *Phys. Lett.* **B324**, 52 (1994).

11. T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi, and Y. Okada, LBL-34796, KEK Preprint 93-146, to appear in *Phys. Rev.* **D**.

12. S. Dimopoulos and A. Pomarol, CERN-TH-95-44, hep-ph/9502397.

13. R. Barbieri and L.J. Hall, *Phys. Lett.* **B338**, 212 (1994); R. Barbieri, L.J. Hall and A. Strumia, IFUP-TH-72-94, hep-ph/9501334; J. Hisano, T.D Moroi, K. Tobe, M. Yamaguchi, and T. Yanagida, TU-476, hep-ph/9501407.

14. S. Dimopoulos and L.J. Hall, *Phys. Lett.* **B344** 185 (1995).

15. A. Antaramian, L.J. Hall, H. Murayama, and A. Rasin, in preparation. See also, H. Murayama and D.B. Kaplan, *Phys. Lett.* **B336**, 221 (1994).